“MECHANICAL PROPERTIES OF MATERIALS”
Overview

1) ELASTIC DEFORMATION
   - Elastic Behavior
   - Anelasticity
   - Elastic Properties of Materials

2) PLASTIC DEFORMATION
   - Tensile Properties
     Yielding and Yield Strength
     Tensile Strength
     Ductility, resilience and toughness
   - Elastic recovery during plastic deformation
   - True Stress and true strain

3) HARDNESS
TESTING OF MATERIALS

Materials, in service, are subjected to forces or loads, e.g. Aluminum alloy in airplane wing, steel automobile axle. It is necessary to know the mechanical properties of such components to design the member from which it is made such that any resulting deformation will not be excessive and fracture will not occur.

There are three principal ways in which a load may be applied to a material: tension, compression, and shear.

Common stress-strain tests:

1) Tension tests
2) Compression tests
3) Shear and torsional tests
1) Tension Test

- Specimen is deformed, usually to fracture, with gradually increasing tensile load.

- The output of such a tensile test is recorded on a strip chart as load or force versus elongation.

- Tensile stress stretches the interatomic distances to lengthen.

* Engineering stress, $\sigma$ (Pa)

\[
\sigma = \frac{F}{A_0}
\]

* Engineering strain, $\varepsilon$ (unitless)

\[
\varepsilon = \frac{l_i - l_0}{l_0} = \frac{\Delta l}{l_0}
\]
2) Compression Test

- Specimen contracts along the direction of the stress.

- By convention, compressive force is taken to be negative, which yields a negative stress. Furthermore, strain is also negative.

- Compressive stress squeezes the atoms closer together to shorten the interatomic distances in metal.
3) Shear Test

**Shear stress** ($\tau$): $\tau = \frac{F}{A_o}$, where $F$ is the force parallel to the surface

**Shear strain** ($\gamma$): defined as the tangent of the strain angle, $\theta$

$\gamma = \tan \theta$
4) Torsional Test

- Torsion is a variation of pure shear, wherein a structural member is twisted like in machine axles and drive shafts.
  - $\phi$: the angle of twist.
  - $\tau$ (shear stress): a function of torque $T$ while $\gamma$ is a function of $\phi$.

- This test is usually performed on cylindrical shafts or tubes.
Tensile Testing of Materials

- Specimens are pulled at constant rate (quasi-static) up to fracture
- Applied load is measured by load cell and length change during testing is measured using extensometer
- Standard test specimens (with rectangular or circular cross-section) are used

![Diagram of a tensile test machine](image)

**Typical tensile test machine**

**Output of a Tensile Testing**

- Load or Force vs Elongation curve

![Force vs Elongation curve](image)
Stress, $\sigma$ (Pa) = Force, $F$ (N) / Initial area, $A_o$ (m$^2$)

Strain, $\varepsilon$ (unitless) = Length change ($\Delta l$) / initial length ($l_o$)

$\varepsilon = (l_i - l_o) / l_o = \Delta l / l_o$

$A_o$: initial cross-sectional area

$l_o$: initial length of the specimen

$A_i$: instantenous cross-sectional area

$l_i$: instantenous length of the specimen

Force vs elongation curve is converted into stress, $\sigma$ and strain, $\varepsilon$ curve
A typical engineering stress-strain curve consists of:

1- Elastic deformation region
2- Plastic (permanent) deformation region
3- Fracture point
1-Elastic Deformation

- Elastic deformation is nonpermanent, which means that when the applied load is released, the piece returns to its original shape.

Elastic means **reversible!**

The deformation in which stress, \( \sigma \) and strain, \( \varepsilon \) are proportional is called **ELASTIC DEFORMATION**
Elastic Deformation on atomic scale

- On an atomic scale, elastic strain is manifested as small changes in the interatomic spacing and the stretching of interatomic bonding.

- The magnitude of the modulus of elasticity is a measure of the resistance to separation of adjacent atoms, that is, the interatomic bonding forces.
Equations used in Elastic Deformation Region

How can you determine change in the dimensions (change in diameter $\Delta d$ ($d_i - d_o$), length $\Delta l$ ($l_i - l_o$)) of specimens in elastic region?

- By use of equations valid in elastic regime only:

- **Hooke’s Law** ($\sigma = E \varepsilon$) Where $\sigma$: uniaxial stress (Pa=N/m$^2$)
  $E$: Elastic or Young’s Modulus
  $\varepsilon$: Strain along tensile axis(or axial strain), ($\Delta l/l_o$)

- **Poisson’s Ratio** $\nu$

  \[
  \nu = - \frac{\varepsilon_X}{\varepsilon_Z} = - \frac{\varepsilon_Y}{\varepsilon_Z}
  \]

  $\varepsilon_X$: lateral strain
  $\varepsilon_Z$: axial strain
Hooke’s Law

- For most metals in the elastic region, stress and strain are proportional to each other through the relationship:

\[ \sigma = E \varepsilon \]

- Hooke’s Law is valid only in the elastic deformation region.

- \( E \) is a measure of material’s stiffness or materials resistance to elastic deformation.
Modulus of elasticity (E) is proportional to the slope of the interatomic force-separation curve at the equilibrium spacing.

$E_{\text{ceramics}} > E_{\text{metals}} > E_{\text{polymers}}$
Poisson's Ratio

- When a tensile load is applied to a material, an elongation in the direction of applied load ($\Delta l_z$), and contractions, ($\Delta l_x$ and $\Delta l_y$) in x and y-directions will appear, respectively.

- **Poisson's ratio, $\nu$:**

  $$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z}$$

  - metals: $\nu \approx 0.33$
  - ceramics: $\approx 0.25$
  - polymers: $\approx 0.40$

- Valid in elastic deformation region
- $\nu < 0.5$
Other Elastic Properties

- Elastic Shear modulus, $G$:
  \[ \tau = G \gamma \]

- Elastic Bulk modulus, $K$:
  \[ P = -K \frac{\Delta V}{V_0} \]
  Pressure test:
  Initial volume $= V_0$
  Volume change $= \Delta V$

- Special relations for isotropic materials:
  \[ G = \frac{E}{2(1 + \nu)} \]
  \[ K = \frac{E}{3(1 - 2\nu)} \]
Elastic deformation will continue after the stress application and during the complete recovery (Time-dependent elastic deformation)

- For **metals** anelastic component is normally small and is usually neglected.
- For **polymers**, its magnitude may be significant (viscoelastic behavior).
2-Plastic (Permenant) Deformation

- As the deformation is proceeded beyond a strain value, the stress is longer proportional to strain, and permanent, non recoverable, or plastic deformation occurs.

- For crystalline materials: plastic deformation is accomplished by means of a process called SLIP. (motion of dislocation)
- In non-crystalline solids deformation occurs by viscous flow

- YIELD STRESS, $\sigma_y$: The stress at which plastic deformation starts

The magnitude of yield strength is a measure of material’s resistance to plastic deformation.
Plastic deformation occurs by means of a process called slip, which involves the motion of dislocations.

Plastic deformation corresponds to the breaking of bonds with original atom neighbors and reforming bonds with new neighbors.
Determination of Yield Strength using Stress-Strain Curves

There are several methods used depending on the type of stress-strain curve;

1) Yield strength is defined as the initial departure from linearity of the stress-strain curve, (Proportional limit, P). It is difficult to determine

2) A straight line is constructed parallel to the elastic portion of the stress-strain curve at a strain value of 0.002. The stress corresponding to the intersection of this line is defined as yield strength, $\sigma_y$.

3) In materials, e.g. Steels, elastic-plastic transition is very well defined and occurs abruptly in what is termed a yield point phenomenon.

4) For materials having non-linear elastic region yield stress is defined as the stress required to produce some amount of strain (e.g. $\varepsilon = 0.005$)
The yield strength is the average of lower limit.

If the transition from elastic to plastic behavior is gradual, the point of yielding may be determined as the initial departure from linearity (P=proportional limit). A straight line is constructed parallel to the elastic deformation line at a strain offset usually 0.002. The stress corresponding to this point is yield strength ($\sigma_y$).
There are materials (gray cast iron, concrete, and many polymers) for which this initial elastic portion of the stress-strain curve is not linear.
Tensile Strength (T.S. or UTS)

- Even though the engineer designs on the basis of yield strength, it is commonly desirable to know the maximum load a metal will support without complete failure.

- What is the reason of decrease in stress subsequent to point M?

  Original area is used in the calculations of stress up to fracture point.
Elastic Recovery during plastic deformation

- Upon release of the load after yielding during testing, some fraction of the total deformation is recovered as elastic strain.

- Upon reloading; yielding will occur at higher stress levels where the unloading began (STRAIN HARDENING)

- There will also be an elastic recovery associated with fracture.
A typical stress-strain curve
Brass (Cu-Zn alloy) specimen
True Stress and Strain Curves

- In engineering stress-strain curves, the cross-sectional area is decreasing rapidly within the neck region, where deformation is occurring. This indicates that the load-bearing capacity of the specimen decreases.

- According to engineering stress-strain diagrams, after point M (UTS), the strength of materials decrease. In fact, the material’s strength increases.

\[ \sigma_T = \frac{F}{A_i} \]

\[ \epsilon_T = \ln \left( \frac{l_i}{l_0} \right) \]

\( \sigma_T \): True stress  
\( \epsilon_T \): True strain  
\( F \): applied load  
\( A_i \): instantaneous cross-sectional area  
\( l_i \): instantaneous length  
\( l_0 \): initial length
* The design engineer always uses the engineering stress-strain curve, because design calculations for engineering products are based on the original dimensions. (In industrial applications true stress and strain are unknown)

**Relationship between Engineering and True Stress-strain**

- **Up to necking, specimen volume remains constant;**

\[ A_i l_i = A_0 l_0 \]

- \( A_i \): instantenous cross-sectional area
- \( l_i \): instantenous length
- \( A_0 \): initial cross-sectional area
- \( l_0 \): initial length

\[ \sigma_T = \sigma (1 + \epsilon) \]
\[ \epsilon_T = \ln (1 + \epsilon) \]

These equations are valid up to the onset of necking, beyond necking actual stress or strain has to be calculated using actual load and area/length.
With the formation of necking, axial stress is no longer axial instead we observe a complex stress state within the neck region. As a result the correct stress (axial) within the neck is slightly lower than the stress computed from the applied load and neck X-sectional area.

The region of true stress and true strain curve from the onset of plastic deformation to the beginning of necking:

\[ \sigma_T = K(\varepsilon_T)^n \]

K and n are constants

Relationship between True Stress and strain up to necking
Other Mechanical Properties

1-Ductility

- It is a measure of the total plastic strain that accompanies fracture.
- % elongation (EL) or % area reduction (AR) is used for measurement of ductility.

\[ \% EL = \frac{L_f - L_o}{L_o} \times 100 \]

or

\[ \% AR = \frac{A_o - A_f}{A_o} \times 100 \]

- Ductility may be expressed quantitatively as either percent elongation or percent area reduction.

\[ L_f \text{ and } A_f \text{ are length and area at the fracture.} \]
### 2-Resilience

- The capacity of a material to absorb energy when it is deformed elastically and then upon unloading to have this energy recovered.

**Modulus of resilience** ($U_r$) = strain energy per unit volume required to stress a material from an unloaded state up to the point of yielding.

\[
U_r = \int_0^{\epsilon_y} \sigma \, d\epsilon
\]

Assuming linear elastic region;

\[
U_r = \frac{1}{2} \sigma_y \epsilon_y
\]

\[
U_r = \frac{1}{2} \sigma_y \epsilon_y = \frac{1}{2} \sigma_y \left( \frac{\sigma_y}{E} \right) = \frac{\sigma_y^2}{2E}
\]

**Units:** Joules/m$^3$ (or, Pa)

Schematic representation showing how modulus of resilience (corresponding to the shaded area) is determined from the tensile stress–strain behavior of a material.
3-Toughness

- It is a measure of the ability of a material to absorb energy up to fracture.

- Approximate by the area under the stress-strain curve.

Units: Joules/m$^3$ (or, Pa)
Example: Stress-Strain Curves
4-HARDNESS

- Measure of a material’s resistance to localized plastic deformation.

- **Hardness measurement:**

  A small indenter is forced into the surface of a material to be tested, under controlled conditions of load and rate of application. The depth or size of the resulting indentation is measured, which in turn related to a hardness number.
HARDNESS TESTS:

1) Rockwell Hardness Testing (HR)
2) Brinell Hardness Testing (HB)
3) Knoop Testing (HK)
4) Vickers Testing (HV)

Macro-hardness testing

Micro-hardness testing

increasing hardness
# Hardness Testing Techniques

## Table of Hardness Testing Techniques

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<td>10-mm sphere of steel or tungsten carbide</td>
<td><img src="image1" alt="Side View" /></td>
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<td>$HB = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$</td>
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<td>Diamond pyramid</td>
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<td>$P$</td>
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<tr>
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<td>$HK = 14.2P/l^2$</td>
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<tr>
<td>Rockwell and Superficial Rockwell</td>
<td>Diamond cone of steel spheres</td>
<td><img src="image4" alt="Side View" /></td>
<td><img src="image5" alt="Side View" /></td>
<td>$\left{ \begin{array}{l} 60 \text{ kg} \ 100 \text{ kg} \ 150 \text{ kg} \ 15 \text{ kg} \ 30 \text{ kg} \ 45 \text{ kg} \end{array} \right.$ Rockwell Superficial Rockwell</td>
</tr>
</tbody>
</table>

$^a$ For the hardness formulas given, $P$ (the applied load) is in kg, while $D, d, d_1,$ and $l$ are all in mm.
Relationship between hardness and strength

$$\sigma_T = 3.5 \text{ BHN}$$

- The hardness test is used as indicator of strength.
- The strength of cast iron, steel and brass can be estimated.
- In brittle materials, e.g. Ceramics, similar correlation between hardness and strength does not occur since cracks may form that serve as stress raisers for propagation of fractures.
**Important Terms:**

* Anelasticity
* Ductility
* Elastic deformation
* Elastic recovery
* Engineering strain
* Engineering stress
* Hardness
* Modulus of elasticity
* Plastic deformation
* Poisson’s ratio
* Proportional limit
* Shear
* Tensile strength
* Toughness
* Yielding
* Yield strength